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Resolution of Closely Spaced Optical Targets Using Maximum Likelihood Estimator and Maximum Entropy Method: A Comparison Study

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FOR THE COMMANDER

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RESOLUTION OF CLOSELY SPACED OPTICAL TARGETS USING MAXIMUM LIKELIHOOD ESTIMATOR AND MAXIMUM ENTROPY METHOD: A COMPARISON STUDY

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Group 22

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ABSTRACT

The capabilities of the maximum likelihood estimator (MLE) and the maximum entropy method (MEM) in resolving closely spaced optical point targets are compared using Monte Carlo simulation results for three different examples. It is found that the MEM is very sensitive to the noise and that the MLE performs better than the MEM in all examples.

CONTENTS

I. INTRODUCTION

Resolving closely spaced objects (CSO) is an important task in optical signal processing. The optical signal is often subject to blurring by a convolutional function and contamination by noise so that the workable frequency bandwidth is limited and hence the resolution of the restored signal is also limited. Several nonlinear processing techniques have been proposed and employed with the aim to increase the resolution power [1]. By incorporating a priori knowledge that the restored optical signal must be positive, those techniques are generally able to extrapolate the signal bandwidth. Among them, the maximum likelihood estimator (MLE) and the maximum entropy method (MEM) are two common approaches.

The well-known MLE has enjoyed a wide application in the area of estimation. It has been shown that MLE is asymptotically unbiased and efficient under certain general conditions. If the noise is additive gaussian then this estimator becomes equivalent to the least-square method. The MEM, which was originally developed to enhance the spectral resolution on short data records [2], is currently still a topic of interest. Besides spectral analysis, MEM has been adopted in a number of other areas where resolution is of great concern.

Early reports seemed to imply the MEM is a method with super resolution power. However, recent studies have indicated that its performance degrades significantly in the presence of noise [3]. In [4], it was suggested that the MEM is superior to the MLE in restoring optical impulse functions although no direct comparison was given.

The objective of this report is to compare the capabilities of MLE and MEM in resolving two optical CSO's. Since an analytical approach to this problem did not appear feasible, the comparison was based on the average characteristics of the Monte Carlo simulation results. Three examples which involved different blurring functions were employed in the simulation study. Of particular interest was the third example which dealt with the CSO resolution problem in the environment of a scanning detector and had been studied in [5] by using MLE. It was found that MLE could actually resolve the closely spaced optical targets better than MEM.

II. RESTORATION OF OPTICAL IMPULSES USING MLE AND MEM

The problem of optical impulse restoration is illustrated by Fig. 1. In many applications, the optical objects are located so far away from the sensor that they can be practically regarded as point targets and represented mathematically by the summation of impulse functions,

$$o(x) = \sum_{i=1}^{m} \alpha_i \, \delta(x-\beta_i) \qquad \alpha_i \geq 0 \tag{1}$$

where m is the number of point targets; α_i and β_i are the intensity and position of the ith target. In optics, α_i is physically constrained to be positive. The observation is assumed to be related to the input by

$$y(x) = o(x) \circledast g(x) + n(x) \tag{2}$$

where g(x) is the blur function and n(x) is an additive white gaussian noise (WGN).* Note that the symbol $\circledast$ denotes convolution. Given the observation, the essential problem is to estiamte $\{\alpha_i\}$ and $\{\beta_i\}$. For the following discussion, we will further assume that m=2 (two targets), that $\int_{-\infty}^{\infty} g^2(x)\,dx=1$ (unit-energy blur function) and that only discrete samples $\{y_i,\ i=1,\ 2,\ldots,N\}$ which are observed at $\{x_i,\ i=1,\ 2,\ldots,N\}$ over the spatial range $[-X,X]$ are available.

The maximum likelihood estimator finds $\hat{\alpha}_i$ and $\hat{\beta}_i$, the estimates

*The WGN assumption is appropriate when the observation noise is determined either by circuit thermal noise or by "shot" noise with very large intensity.

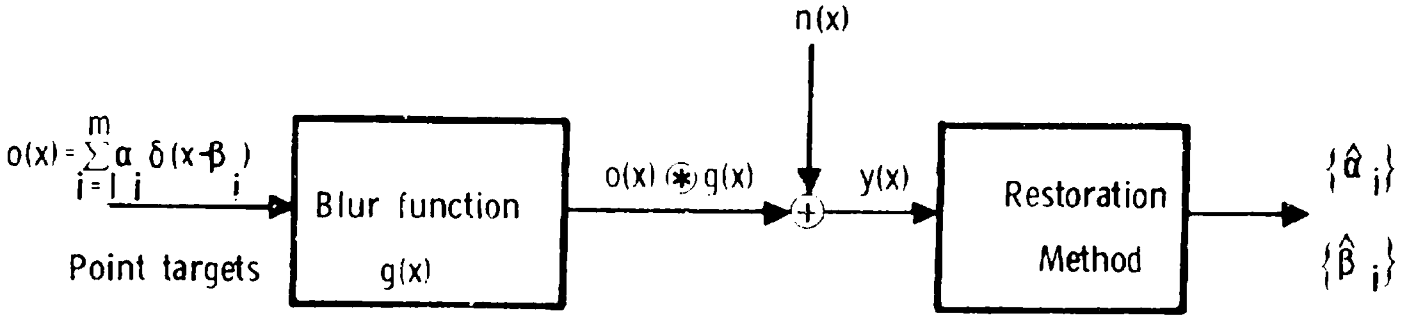


Fig. 1. Problem of optical impulse restoration.

of α_i and β_i by maximixing the likelihood function which is given by

$$\ell(\underline{\alpha},\underline{\beta}) = \frac{1}{(2\pi)^{N/2}\sigma_n^N} \exp\left\{-\frac{1}{2\sigma_n^2}(Y-G\underline{\alpha})^T(Y-G\underline{\alpha})\right\} \tag{3}$$

where $\underline{\alpha}=(\alpha_1,\alpha_2)^T$, $\beta=(\beta_1,\beta_2)^T$, $Y=(y_1,y_2,\ldots,y_N)^T$ and

$$G = \begin{pmatrix} g(x_1-\beta_1) & g(x_1-\beta_2) \\ \vdots & \vdots \\ g(x_N-\beta_1) & g(x_N-\beta_2) \end{pmatrix} \tag{4}$$

σ_n is the standard deviation of the WGN. It can be shown that [3]

$$\hat{\underline{\alpha}} = (G^TG)^{-1}G^TY \tag{5}$$

and $\hat{\underline{\beta}}$ satisfies the following equation,

$$L(\hat{\underline{\beta}}) = \max_{\beta_1,\beta_2} L(\beta_1,\beta_2) \tag{6}$$

where

$$L(\beta_1,\beta_2) = Y^TG(G^TG)^{-1}G^TY. \tag{7}$$

By defining

$$\kappa(\beta_i) \triangleq \sum_{j=1}^{N} g(x_j-\beta_i)\ y_j \quad i=1,2 \tag{8}$$

$$\rho(\beta_1,\beta_2) = \sum_{j=1}^{N} g(x_j-\beta_1)\ g(x_j-\beta_2) \tag{9}$$

we can rewrite (7) as

$$L(\underline{\beta}) = \frac{1}{1-\rho^2}\left\{\kappa^2(\beta_1) + \kappa^2(\beta_2) - 2\rho\,\kappa(\beta_1)\kappa(\beta_2)\right\}. \tag{10}$$

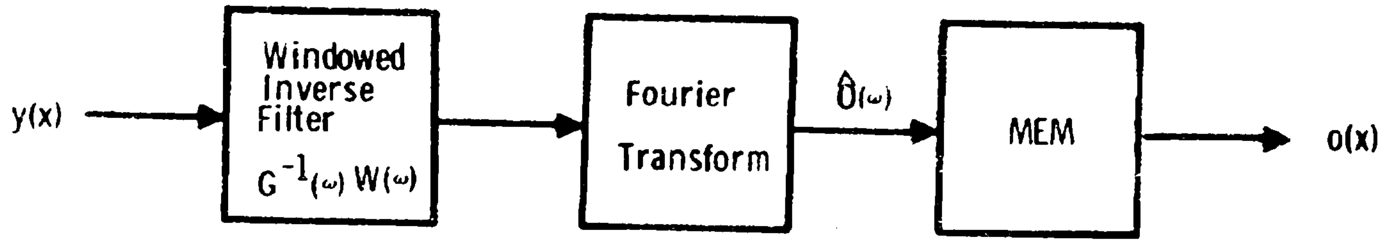
To obtain the optimal solutions for $\hat{\underline{\alpha}}$ and $\hat{\underline{\beta}}$ using (5) and (6) in conjunction with (7) or (10) usually requires a constrained nonlinear optimization algorithm which is numerically complicated and computationally costly. A suboptimal solution is available if we restrict $\hat{\beta}_1$ and $\hat{\beta}_2$ to be coincident with the sampling positions [6]. In this case, $\hat{\underline{\beta}}$ can be found by searching for the global maximum of (10) in a double do-loop fashion over $\{(x_i, x_j)\}$ where the indices are such that $1 \leq i < j \leq N$ and the corresponding $\hat{\alpha}_i$ and $\hat{\alpha}_j$ must be both positive. During the search, the required $\kappa(\hat{\beta}_i)$, $i=1,2$ and $\rho(\hat{\beta}_1, \hat{\beta}_2)$ can be obtained simply by looking up the precomputed and stored output sequences of the discrete matched filter and the discrete autocorrelator respectively. If the optimal solution is desired the output of this suboptimal approach can be fed into a nonlinear optimization algorithm such as the Quasi-Newton method as the initial guess.

The maximum entropy method for restoring the optical impulse function is shown in Fig. 2. The MEM, which is a nonparametric approach, finds $\hat{o}(x)$, the restored signal, by maximizing its entropy

$$H = \int_{-X}^{X} \ln \hat{o}(x)dx \tag{11}$$

while satisfying the constraint

$$\hat{O}(\omega_k) = \int_{-X}^{X} \hat{o}(x)\, e^{-j\omega_k x} dx. \tag{12}$$



$$W(\omega) = \begin{cases} 1 & |\omega| \leq \Omega_c \\ 0 & \text{otherwise} \end{cases}$$

Fig. 2. MEM as applied in restoring optical impulsive functions.

$\hat{O}(\omega_k)$ is obtained from the windowed inverse filter.

The choice of the window width, Ω_c, of the inverse filter represents a compromise between spatial resolution and signal-to-noise ratio (SNR). On one hand the window should be made as wide as possible in order to provide higher spectral resolution. On the other hand, it should be kept as narrow as possible in order to eliminate the noise effect. The idea of the MEM is to extrapolate, in accordance with the principle of maximum entropy, the bandwidth of the signal at the output of the inverse filter which has limited the signal bandwidth to the region where SNR is relatively high.

The solution of (11) and (12) has been shown to be given by [7]

$$\hat{o}(x) = \frac{(2X)^{-1}\gamma_{K+1}}{\left|1 + \sum_{k=1}^{K} \gamma_k \exp\left\{-jk\pi(2X)^{-1}(x+X)\right\}\right|^2} \qquad -X \leq x \leq X \tag{13}$$

with the K+1 unknowns $\{\gamma_k\}$ determined as the solution of the K+1 Yule-Walker linear equations

$$
\begin{pmatrix}
\hat{O}'(0) & \hat{O}'(\omega_K) & \cdot\ \cdot & \hat{O}'(\omega_K) \\
\hat{O}'(\omega_1) & \hat{O}'(0) & \cdot\ \cdot & \hat{O}'(\omega_{K-1}) \\
\cdot & & & \\
\cdot & & & \\
\hat{O}'(\omega_K) & \hat{O}'(\omega_{K-1}) & \cdot\ \cdot & \hat{O}'(0)
\end{pmatrix}
\begin{pmatrix} 1 \\ \gamma_1 \\ \cdot \\ \cdot \\ \gamma_K \end{pmatrix}
=
\begin{pmatrix} \gamma_{K+1} \\ 0 \\ \cdot \\ \cdot \\ 0 \end{pmatrix}
\tag{14}
$$

where $\omega_k = k\pi(2X)^{-1}$, $k=0,1,\ldots,K$ and

$$
\hat{O}'(\omega_k) = 2R_e\left\{\hat{O}(\omega_k)\ \exp\ (-j\omega_k X)\right\} \tag{15}
$$

Note that the width of the window of the inverse filter is effectively determined by the parameter K and is equal to $1/2K(2X)^{-1}$. Equation (14) can be solved recursively by using the Levinson algorithm [8]. The spatial peaks of $\hat{o}(x)$ can be identified by locating the minima in the denominator of (13).

III. COMPARISON VIA SIMULATIONS

Three examples were used to compare the resolution capabilities of the MLE and the MEM. Here the resolution capability was evaluated only in terms of parameter estimation performance although the detection performance should also be considered. We assume that a priori knowledge of the number of targets (detection problem) was available. Since we were more concerned with the positions of the targets, we compared the two methods in terms of the performance (standard deviation and bias) of position estimation.

For the purpose of this comparison, the suboptimal implementation of the MLE as described in the previous section was employed in the first two examples and the optimal approach in the third example. The estimation statistics were collected from 100 Monte Carlo runs. The procedure for the MEM was more complicted because the output depended on the order of the set of linear equations, (14), and the number of minima exhibited in the denominator of (13) was not always exactly equal to 2, the number of targets. For each order of equations, the first 100 Monte Carlo runs which displayed two or more minima were used in computing the statistics. For each of those runs, the locations of two lowest minima were taken as the position estimates of the two targets. The best estimation performance (minimum estimation variances) among all orders of equations examined was considered as the performance of the MEM.

For each of the three examples, the separation Δ_x between the two optical targets as well as the SNR were varied. The SNR was defined as

$$\mathrm{SNR}_i = \left[\sum_{j=1}^{N} (\alpha_i g(x_j))^2 \right]^{\frac{1}{2}} / \sigma_n \qquad (16a)$$

$$\cong \alpha_i / (\sigma_n \sqrt{\delta_x}) \qquad i=1,2 \qquad (16b)$$

where δ_x is the spatial sampling step. In all cases, SNR_1 and SNR_2 were set equal and the two targets were located at $\beta_1, \beta_2 = \pm\Delta_x/2$. Since the configuration of the targets was symmetrical and the blur functions used in the examples were even functions, the standard deviations as well as biases of $\hat{\beta}_1$ and $\hat{\beta}_2$ were expected to be asymptotically equal. Therefore only the average value was used to characterize the estimation accuracy.

Example 1:

$$\begin{cases} g(x) = (\pi\sigma^2)^{-\frac{1}{4}} \exp[-\frac{1}{2}(x/\sigma)^2], \ \sigma = 10 \\ N \text{ (sample size)} = 128 \\ \delta_x = 1 \end{cases} \qquad (17)$$

The blur function in this example which is similar to the one used in [4] is usually observed when atmospheric turbulence is significant. The associated autocorrelation function $\rho(\tau)$ and Fourier transform $G(\omega)$ are given by

$$\rho(\tau) = \exp - \left[\left(\frac{\tau}{2\sigma}\right)^2\right] \tag{18}$$

$$G(\omega) = (4\pi\sigma^2)^{\frac{1}{4}} \quad \exp(-\tfrac{1}{2}\sigma^2\omega^2) \tag{19}$$

The performances of the MLE and the MEM are compared as shown in Fig. 3. The standard deviation σ_x and the bias b_x of the position estimation are plotted against the target separation Δ_x for SNR = 20 and 10. For either of these two methods, it can be observed that the performance degrades when Δ_x becomes smaller or when SNR is reduced. However when compared with each other, it is obvious that the MLE is better than the MEM in the sense of smaller σ_x and b_x for almost all combinations of Δ_x and SNR.

From Fig. 3, it can also be seen that the two targets behave like isolated targets for larger Δ_x(e.g., $\geq 3\sigma$) where σ_x and b_x remain almost constant. If the effect of δ_x (equal to 1) on the estimation accuracy is taken into consideration, it is acceptable to say, based on data from Fig. 3, that in this separation range either MEM or MLE is almost unbiased ($b_x \leq \delta_x$) and their standard deviations are close to the square root of the Cramer-Rao bound* for a single target which is equal to .707 for SNR=20 and 1.414 for SNR= 10.

*The Cramer-Rao bound (CRB) is the lower bound on the variance of an unbiased estimator. For a single target, the CRB of the position estimate is equal to $1/(\mathrm{SNR}\cdot B)^2$ where B defined as $B^2 = \int_{-\infty}^{\infty} \omega^2 G^2(\omega)\,d\omega / \int_{-\infty}^{\infty} G^2(\omega)\,d\omega$ is the so-called root-mean-square bandwidth. In example 1, B=.0707.

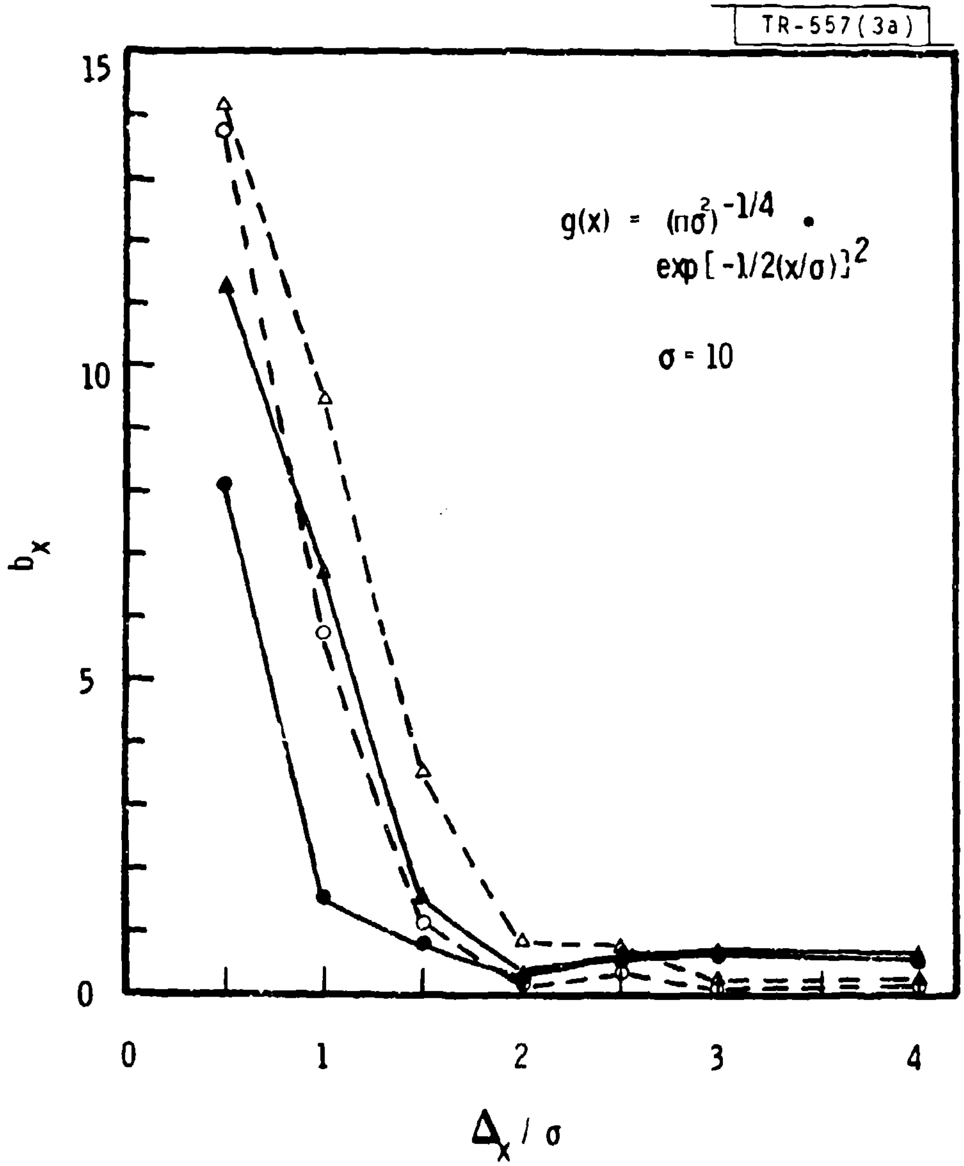


Fig. 3(a). Performance comparison of MLE and MEM, example 1. Estimatimation bias, b_x, is plotted as a function of target separation, Δx; Δ_x is normalized with σ of $g(x)$.

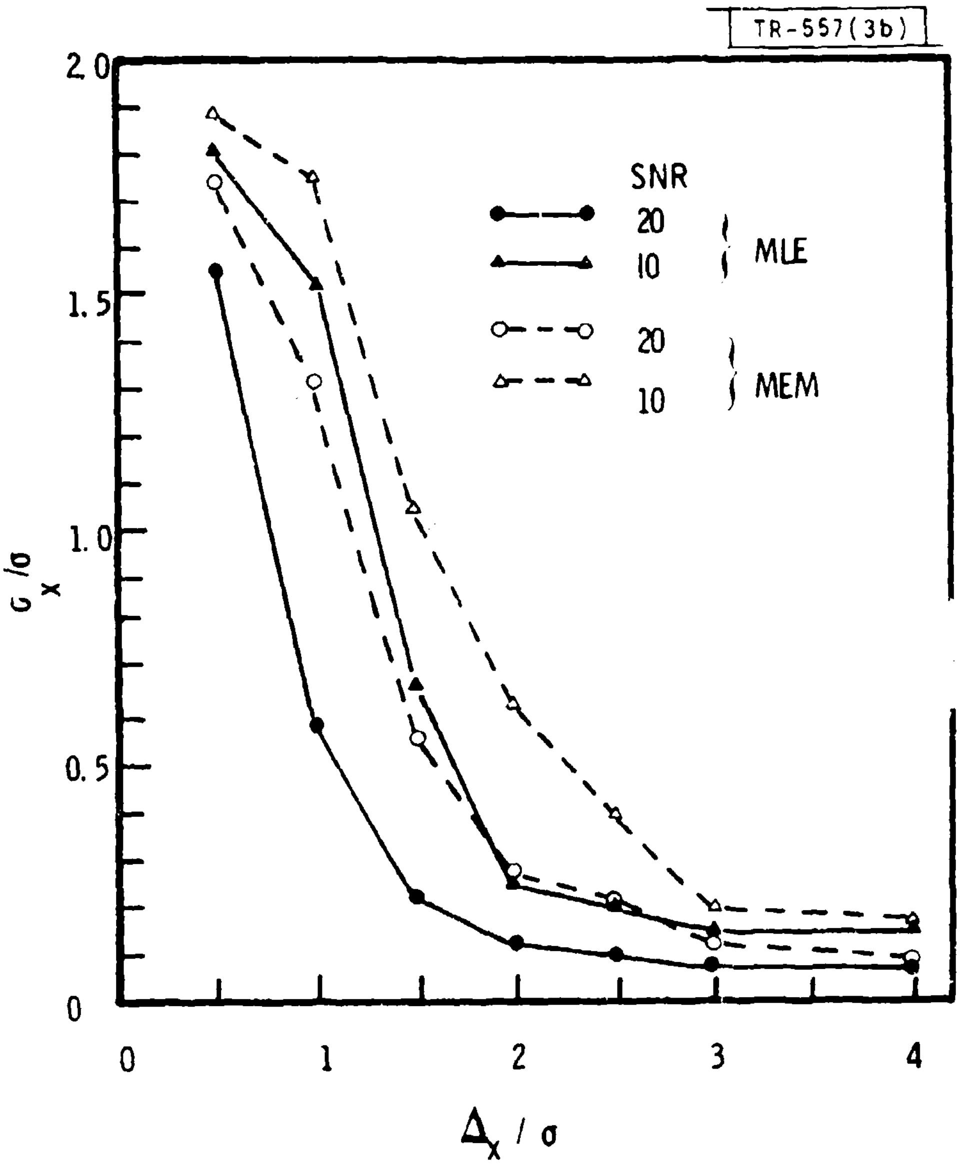


Fig. 3(b). Performance comparison of MLE and MEM, example 1. Estimation standard deviation, σ_x, is plotted as a function of target separation Δ_x. Both σ_x and Δ_x are normalized with σ of g(x).

Example 2:

$$
\begin{cases}
g(x) = (3/2)^{1/2} \sin^2(\pi x)/(\pi x)^2, \quad x=\theta/(\lambda/D) \\
N=128 \\
\delta_x=.1
\end{cases}
\tag{20}
$$

Here g(x) is the blur function due to a slit aperture with length equal to D. The classical Rayleigh resolving power in angle (θ) for this type of aperture is equal to λ/D where λ is the wavelength of the incident light. The associated $\rho(\tau)$ and $G(\omega)$ are given in the following:

$$
\rho(\tau) = \frac{2\pi\tau - \sin(2\pi\tau)}{2(\pi\tau)^3}
\tag{21}
$$

$$
G(\omega) = \begin{cases}
1 - \dfrac{|\omega|}{2\pi}, & |\omega| \leq 2\pi \\
0 & \text{otherwise}
\end{cases}
\tag{22}
$$

Note that this blur function is bandlimited.

Figure 4 shows the comparison of the MLE and the MEM using this example with SNR=20. Obviously, the MLE is much better than the MEM in estimating the positions of the optical CSO's. Note that the hump of the MLE σ_x curve is due to the first sidelobe of g(x).

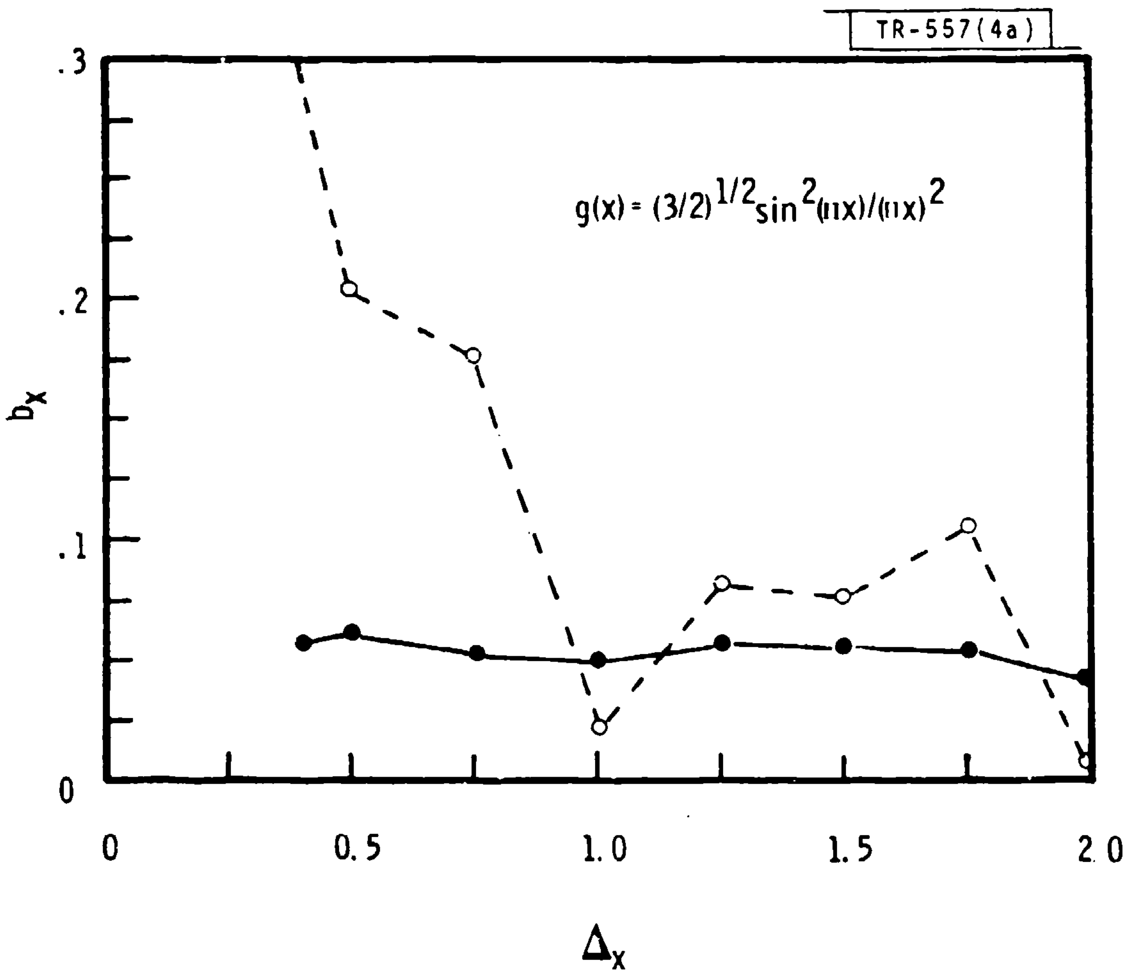


Fig. 4(a). Performance comparison of MLE and MEM, example 2. Estimation bias, b_x, is plotted as a function of target separation, Δ_x.

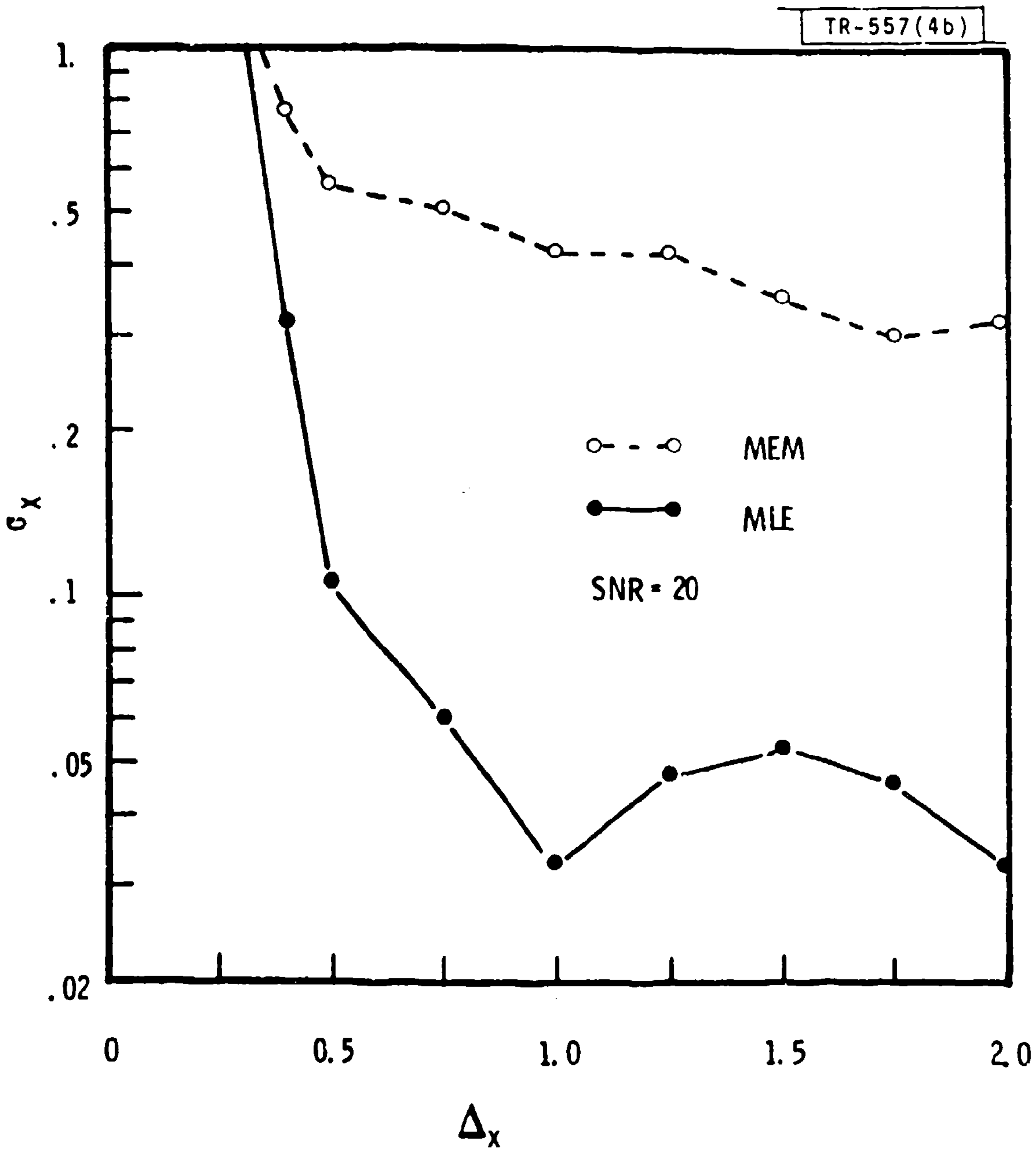


Fig. 4(b). Performance comparison of ⌄⌄.E and MEM, example 2. Estimation standard deviation, σ_x, is .lotted as a function of target separation Δ_x.

Example 3:

This example deals with the CSO resolution in the environment of a scanning detector. The problem has been treated in [5] using the MLE. A brief description is as follows. A detector scans through the focal plane of an optical sensor on which an image due to a pair of CSO's in the field-of-view of the sensor has been produced. The spatial structure of the image is thus converted into a temporal signal. Without loss of generality, a unity scanning rate can be assumed so that the spatial and time coordinates are equivalent. It is also assumed that the two CSO's are lying in the scan direction (e.g., x-axis). The blur function g(x) appearing in the signal model, Eq. (2), is now given by the convolution of the point spread function p(x,y) and the detector response function d(x,y). Here in this example, p(x,y) is due to an annular aperture with diameter equal to 2 λ/D (λ/D is the optical diffraction limit) and obscuration coefficient equal to 50%, and d(x,y) is assumed to be uniform over the detector surface whose dimensions are 2 λ/D in scan direction and 6 λ/D in cross-scan direction.

In this example, no closed form expressions can be obtained for g(x) and $G(\omega)$ and therefore they are computed numerically. Sixty-four samples taken at .2 λ/D interval are used.

In Fig. 5, the root-mean-square (RMS) errors, e_x, of the position estimation using MLE and MEM are plotted as functions of

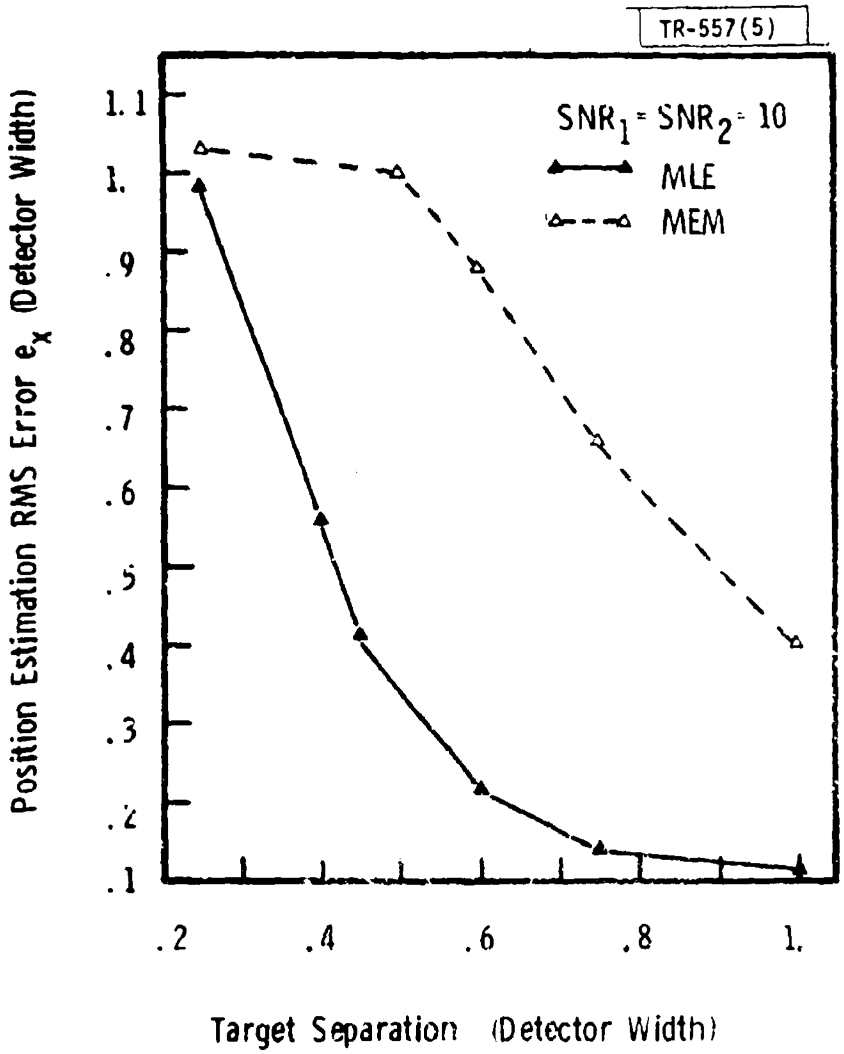


Fig. 5. Performance comparison of MLE and MEM, example 3; Position estimation RMS error as function of target separation.

the target separation. Both variables are expressed in terms of the detector width in the scan direction. Note that $e_x = (b_x^2 + \sigma_x^2)^{\frac{1}{2}}$. Again, it is obvious that the MLE can resolve the CSO's much better than the MEM in this example.

IV. CONCLUSION

Based on simulation results with three diverse examples, it can be concluded that the maximum entropy method, as compared to the maximum likelihood estimator, is not an outstanding approach for increasing the resolution of two closely spaced optical point targets particularly in the presence of noise. The difficulty with the MEM lies in the fact that it is derived without considering the measurement noise and it can only depend on the windowed inverse filter to reduce the noise effect. Although some modifications including the noise compensation scheme [9] and another maximum entropy formulation [10] have been proposed to deal with the noise, the results are still not very convincing. Besides, the simplicity of the MEM has been lost in these new approaches.

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The capabilities of the maximum likelihood estimator (MLE) and the maximum entropy method (MEM) in resolving closely spaced optical point targets are compared using Monte Carlo simulation results for three different examples. It is found that the MEM is very sensitive to the noise and that the MLE performs better than the MEM in all examples.

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